

# Josephson-phase qubit without tunneling

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We show that a complete set of one-bit gates can be realized by coupling the two logical states of a phase qubit to a third level (at higher energy) using microwave pulses. Thus, one can achieve coherent control without invoking any tunneling between the qubit levels. We propose two implementations, using rf-SQUIDs and *d*-wave Josephson junctions.

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In the field of Josephson qubits,<sup>1</sup> phase qubits enjoy continued attention. This is partly due to their tolerance to decohering background noise compared to charge qubits. Most phase-based designs rely on the tunnel splitting  $\Delta$  to flip the state, i.e., to perform a  $\sigma_x$  operation. This has several disadvantages. First,  $\Delta$  is exponentially sensitive to the device parameters. This makes manufacturing spread especially severe, hampering scalability. Second, it is hard to stop the evolution, so one may need to e.g. refocus.<sup>2</sup> One can in principle switch off  $\Delta$  using a compound (Bloch-transistor) junction, but this considerably increases the parameter sensitivity even further.<sup>3</sup> Also, for many systems  $\Delta$  is too small to be useful or even observable. Conversely, current-biased “large-junction” qubits<sup>4,5</sup> avoid the reliance on tunneling at the price of a large spacing between the logical levels, leading to a strong always-on  $\sigma_z$  evolution.

Recently, in Ref. 6 it has been proposed to flip the state of a qubit by two consecutive microwave pulses. The first pulse excites the qubit from, say,  $|0\rangle$  to a higher<sup>7</sup> auxiliary state  $|2\rangle$  through a Rabi oscillation. The next takes the qubit back to the logical space, but now to  $|1\rangle$ , addressing the first disadvantage above. However, this pulse sequence would carry  $|1\rangle$  to  $|2\rangle$  instead of the desired  $|0\rangle$ ; *a fortiori*, it thus does not map a general (superposition) qubit state to another, hence is not a valid gate operation. Even if this would be remedied [by, e.g., preceding (following up) the sequence with an extra  $|1\rangle \leftrightarrow |2\rangle$  ( $|0\rangle \leftrightarrow |2\rangle$ ) pulse], the method’s state selectivity relies on a bias between  $|0\rangle$  and  $|1\rangle$ , so the second disadvantage is overcome at best partially (the bias can be removed during idle periods, but not during gate action); also, the inflexible restriction to bit-flip gates only remains.

Simultaneously, Ref. 8 has given a largely<sup>9</sup> correct proposal of using an auxiliary state to implement some gates for a different class of qubits. The coupling to the third state does not involve microwaves, and the resulting lack of tunability seems to limit the proposal to a discrete set of gates. In this paper, we resolve the abovementioned problems by showing that a general quantum gate *can* be realized with Rabi pulses alone, without using tunneling.

Consider a general system with a bistable potential (Fig. 1). The lowest levels in the left and right wells are taken as the logical states  $|0\rangle$ ,  $|1\rangle$ . Unlike most other phase-qubit designs, we choose our parameters so as to make  $\Delta$  smaller than all relevant energy scales, in par-

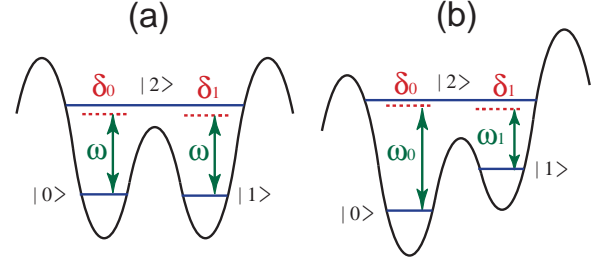


FIG. 1: Gate operation by coupling the two logical states to a third level with microwaves. (a) (Near-)degenerate states and one pulse frequency. (b) Biased states and two frequencies.

ticular the decoherence rate:  $\Delta \ll 1/\tau_\varphi$  ( $\hbar = 1$ ). Then, one can consider  $|0\rangle$ ,  $|1\rangle$  as energy eigenstates.

We induce transitions to a higher state  $|2\rangle$  by applying microwaves with frequencies near the energy differences  $E_2 - E_{0,1}$  (Fig. 1). The system then undergoes Rabi oscillations, starting from the logical space. After half a Rabi period ( $t_c = \pi/\Omega_R$ ), the probability of finding the system in  $|2\rangle$  will be zero again. The qubit wave function, however, will in general have changed: if, e.g., the system starts from  $|0\rangle$ , it will end up in a superposition. Thus, a matrix element has been created between  $|0\rangle$  and  $|1\rangle$ , equivalent to a  $\sigma_x$  term in the reduced Hamiltonian.

More quantitatively, let us write the Hamiltonian as

$$H = H_0 + V(t), \quad (1)$$

$$H_0 = E_0|0\rangle\langle 0| + E_1|1\rangle\langle 1|, \quad (2)$$

where  $H_0$  accounts for the uncoupled Josephson junction (with  $E_2 \equiv 0$  for convenience) and  $V(t)$  for the microwaves. This simple model captures the physics outlined above in two regimes. *Regime (a)* corresponds to near-degenerate logical levels and one external frequency,

$$V(t) = V e^{-i\omega t} + \text{h.c.} \quad (3a)$$

and  $|\delta_j| \lesssim |V|$ , with the detunings  $\delta_j \equiv \omega + E_j$  ( $j = 0, 1$ ) and  $|V|$  the size of a typical matrix element of  $V$  (Fig. 1a). On the other hand, in *regime (b)* the logical levels are well-separated,  $|E_1 - E_0| \gg |V|$ , and each is coupled to  $|2\rangle$  by its own frequency:

$$V(t) = V_0 e^{-i\omega_0 t} + V_1 e^{-i\omega_1 t} + \text{h.c.} \quad (3b)$$

Again  $|\delta_j| \lesssim |V|$ , with now  $\delta_j \equiv \omega_j + E_j$  (Fig. 1b).

We expand the wave function as  $|\psi\rangle = \sum_{j=0}^2 c_j |j\rangle$  and introduce  $\tilde{c}_j = c_j e^{-i\omega_j t}$  ( $j = 0, 1$ ). In the rotating-wave approximation<sup>10</sup> (RWA),  $\tilde{\psi} = (\tilde{c}_0, \tilde{c}_1, c_2)^T$  then obeys  $id_t \tilde{\psi} = \tilde{H} \tilde{\psi}$  with the time-independent<sup>11</sup>

$$\tilde{H} = \begin{pmatrix} \delta_0 & 0 & u_0^* \\ 0 & \delta_1 & u_1^* \\ u_0 & u_1 & 0 \end{pmatrix}, \quad (4)$$

in terms of the only relevant matrix elements  $u_j = \langle 2|V_j|j\rangle$ . In particular, setting  $\omega_0 = \omega_1 \equiv \omega$  and  $V_0 = V_1 \equiv V$  in regime (a), (4) holds in both regimes.

A watershed now occurs between the case of equal detunings<sup>12</sup>  $\delta_0 = \delta_1$ , which will shortly be reduced to the standard Rabi problem, and the more complicated  $\delta_0 \neq \delta_1$ , which however does not correspond to a useful gate operation. Note that in regime (a), the former case is the simple one of degenerate qubit levels; this may be the preferred mode of operation in practice.

First taking  $\delta_0 = \delta_1 = \delta$ , we define the Rabi frequency

$$\Omega_R = \sqrt{\delta^2/4 + |u_0|^2 + |u_1|^2}, \quad (5)$$

and a mixing angle  $0 < \eta < \pi$  by  $\tan \eta = 2\sqrt{|u_0|^2 + |u_1|^2}/\delta$ . One readily finds the inert eigenfunction

$$\tilde{\psi}_0 = \frac{(u_1, -u_0, 0)^T}{\Omega_R \sin \eta} \quad (6)$$

obeying  $\tilde{H} \tilde{\psi}_0 = \tilde{\nu}_0 \tilde{\psi}_0$  with  $\tilde{\nu}_0 = \delta$ , which is decoupled from  $|2\rangle$  by destructive interference of the microwaves.<sup>13</sup> In the complementary  $2 \times 2$  space, simple algebra yields the rest of the spectrum as  $\tilde{\nu}_{\pm} = \Omega_R (\cos \eta \pm 1)$ ,

$$\tilde{\psi}_+ = \left( \frac{u_0^*}{2\Omega_R \sin \eta/2}, \frac{u_1^*}{2\Omega_R \sin \eta/2}, \sin \eta/2 \right)^T, \quad (7)$$

$$\tilde{\psi}_- = \left( \frac{u_0^*}{2\Omega_R \cos \eta/2}, \frac{u_1^*}{2\Omega_R \cos \eta/2}, -\cos \eta/2 \right)^T. \quad (8)$$

In terms of (6)–(8), it is trivial to compute the evolution over half a Rabi period  $\tilde{U}(t_c) = \exp\{-i\tilde{H}t_c\}$ , decomposing into a reduced gate action  $\tilde{U}_r$  in the logical space and a trivial phase for  $|2\rangle$  [cf. above (1) and Figs. 2a,b]. Only the former concerns us here,

$$\tilde{U}_r = \frac{1}{\Omega_R^2 \sin^2 \eta} \begin{pmatrix} \zeta |u_0|^2 + \zeta^2 |u_1|^2 & u_0^* u_1 (\zeta - \zeta^2) \\ u_0 u_1^* (\zeta - \zeta^2) & \zeta^2 |u_0|^2 + \zeta |u_1|^2 \end{pmatrix}, \quad (9)$$

a central result, with  $\zeta = -e^{-\pi i \cos \eta}$  running through the unit circle with detuning. Clearly,  $\tilde{U}_r$  is unitary, overcoming the problem<sup>6</sup> mentioned in the introduction. The repeated evolution  $\tilde{U}(nt_c)_r = \tilde{U}_r^n$  follows by simply putting  $\zeta \mapsto \zeta^n$  in (9); hence, the only advantage of taking  $n > 1$  seems to lie in accessing  $\zeta^n \approx 1$  without large detuning.

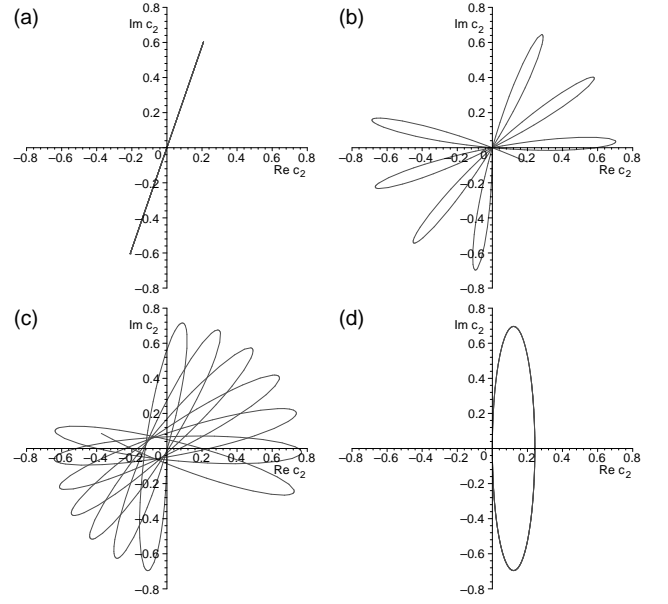


FIG. 2: Evolution of  $c_2(t) \equiv \langle 2|\tilde{U}(t)|0\rangle$  for: (a)  $\delta_0 = \delta_1 = 0$ ,  $u_0 = e^{-i/3}$ ,  $u_1 = 1.2$ ,  $0 \leq t \leq 5\pi$ ; (b)  $\delta_0 = \delta_1 = 0.25$ ,  $u_0 = u_1 = 1$ ,  $0 \leq t \leq 5\pi$ ; (c)  $\delta_0 = 0.25$ ,  $\delta_1 = 0$ ,  $u_0 = u_1 = 1$ ,  $0 \leq t \leq 10\pi$ ; (d)  $-\delta_0 = \delta_1 = 0.25$ ,  $u_0 = 1$ ,  $u_1 = e^{i/3}$ ,  $0 \leq t \leq 5\pi$ .

Let us demonstrate that already in its two simplest limits, (9) is flexible enough to lead to universal computing; contrast Refs. 6,8. For unbiased systems with symmetric potential [cf. (12), (13) below] and  $u_0 = u_1$ ,

$$\tilde{U}_{r,\text{sy}}\left(\frac{\delta}{\Omega_R}\right) = \exp\left\{i\left[\frac{\pi}{2} - \frac{3\pi\delta}{4\Omega_R} + \left(\frac{\pi}{2} + \frac{\pi\delta}{4\Omega_R}\right)\sigma_x\right]\right\}. \quad (10)$$

One can also drive at resonance  $\delta = 0$ , but with arbitrary  $|u_0/u_1|$  [in regime (b)]. Setting  $(|u_0|^2 - |u_1|^2)/(|u_0|^2 + |u_1|^2) = \cos \xi$ , one has  $2u_0^* u_1/(|u_0|^2 + |u_1|^2) = e^{i\gamma} \sin \xi$ , and

$$\tilde{U}_{r,\text{res}}(\xi) = ie^{i(\pi/4 + \gamma/2)\sigma_z} e^{i\xi\sigma_x} e^{i(\pi/4 - \gamma/2)\sigma_z}. \quad (11)$$

Of course, one always has the phase shifts  $e^{i\chi\sigma_z}$  available, by applying a small bias but no microwaves. Thus, the equivalence above (1) is quantitative: adding either (10) or (11) suffices to generate all one-bit gates. For  $u_0 = u_1$ ,  $\delta = 0$ , both of the above reduce to a quantum NOT  $\tilde{U}_r \propto \sigma_x$ ; in general,  $[\tilde{U}_r, \sigma_z] \neq 0$  unless  $u_0 u_1 = 0$ .

In the “laboratory frame”  $\psi_r = (c_0, c_1)^T$ ,  $U_r \propto e^{i(\omega_0 - \omega_1)t_c \sigma_z/2} \tilde{U}_r = e^{i(E_1 - E_0)t_c \sigma_z/2} \tilde{U}_r$  [ $= \tilde{U}_r$  in regime (a)]. For this specific form, it is assumed that the gate operation starts at  $t = 0$ ; this fixes the phases of  $V_{0,1}$  in (3b).

The effective operation rate  $\Omega_R$  in (5) depends on *intra*well matrix elements  $u_j$ , between wave functions having an overlap of  $O(1)$ . For reasonable microwave powers, one thus expects a speedup compared to conventional designs relying on a small  $\Delta$ . Indeed, the analysis of Ref. 6 applies, showing that the number of operations achievable in  $\tau_\varphi$  is increased by an order of magnitude. If anything, the present situation is slightly more favorable still, since our gate operation is a *one*-step process.

Generalizing the above to  $\delta_0 \neq \delta_1$  would lead to tedious cubic equations. Fortunately however, this is unnecessary since the crucial decomposition of  $\tilde{U}(t)$  then does not generally occur for any finite  $t$ .<sup>14</sup> To see this, start e.g. from  $|0\rangle$  and plot  $c_2(t)$  by diagonalizing a few instances of (4) numerically. The locus of  $c_2$  will evolve in a daisy-like pattern (Fig. 2c), without returning to the origin like it does periodically for equal detunings (Figs. 2a,b). These numerics can be supplemented with an expansion in  $\delta_0 - \delta_1$ , the case  $\delta_0 = \delta_1 = 0$  being a particularly simple zeroth-order problem.

Some idealizations have been made in the above:  $H$  as in (1), (2) is a low-dimensional approximation and the effective (4) follows only in the RWA. The pertinent errors typically are  $\sim |V|/|\Delta E|$ , where  $\Delta E$  can be the distance  $E_2 - E'$  (positive or negative) to an ignored level  $E'$  or  $E_1 - E_0$  in regime (b), etc. These can be reduced using a narrow-band, low-power source, but only under the condition  $\Omega_R \tau_\phi \gg 1$  of fast gate operation. The issue is well-understood, and techniques such as pulse-shaping exist to counteract off-resonant (including counter-rotating) errors,<sup>15</sup> in addition to general quantum error-correction methods. The same holds for timing errors.

We now propose two exemplary implementations.

1. *SQUIDS*. One can use any SQUID qubit, such as the three-junction<sup>16</sup> or the usual rf-SQUID. The latter consists of a superconducting ring interrupted by a junction with Josephson energy  $E_J$ . The free energy is

$$\mathcal{U}(\phi) = \frac{(\Phi_0 \phi / 2\pi - \Phi_e)^2}{2L} - E_J \cos \phi, \quad (12)$$

with  $\phi$  the phase difference across the junction and  $\Phi_0 = \pi/e$  the flux quantum. When the external flux  $\Phi_e = \Phi_0/2$  and the ring inductance  $L > \Phi_0^2/4\pi^2 E_J$ ,  $\mathcal{U}$  will have the bistable shape of Fig. 1a. The states  $|0\rangle$  and  $|1\rangle$  correspond to opposite directions of persistent current.

A deviation of  $\Phi_e$  from  $\Phi_0/2$  tilts  $\mathcal{U}$  (Fig. 1b), generating a  $\sigma_z$  operation; applying an rf flux performs a  $\sigma_x$ -like gate (9). To read out the qubit one should measure the SQUID-generated flux at  $\Phi_e = \Phi_0/2$ ; its two directions correspond to the logical states.

2. *Current-biased d-wave junctions*. In *d*-wave grain boundaries, the order parameter is oriented differently on the two sides of the junction. The resulting Josephson potential is intrinsically bistable,<sup>2,17–19</sup> realizing Fig. 1.

In general, the current-phase relation can have many harmonics. Here, we approximate  $I(\phi) = I_1 \sin \phi - I_2 \sin 2\phi$ , where  $I$  is the current through and  $\phi$  the phase difference across the junction. The free energy thus is

$$\mathcal{U}(\phi) = -E_J \left[ \cos \phi - \frac{\alpha}{4} \cos(2\phi) \right] - \frac{I_b}{2e} \phi, \quad (13)$$

where  $E_J = I_1/2e$  is the Josephson energy corresponding to the first harmonic,  $\alpha = 2I_2/I_1$ , and  $I_b$  is the bias current. When  $I_b = 0$ , the minima of (13) are located at

$$\phi = \begin{cases} \pm \arccos(1/\alpha), & \alpha > 1; \\ 0, & \alpha \leq 1. \end{cases} \quad (14)$$

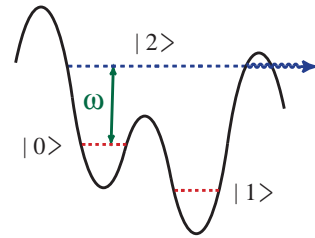


FIG. 3: Qubit readout using microwave-assisted tunneling to the resistive state. Only the left state will tunnel out.

For  $\alpha > 1$ ,  $\mathcal{U}$  thus is doubly degenerate, with barrier height  $\delta\mathcal{U} = E_J(\alpha + \alpha^{-1} - 2)/2$  between the minima.

A finite  $I_b$  removes the degeneracy; this can be used for the  $\sigma_z$  operation. The gate (9) can be performed using ac bias currents with appropriate frequencies, as discussed before. For readout, we apply an  $I_b$  such that one of the excited states has a high probability of tunneling to the continuum (Fig. 3). By selectively coupling one logical state to this excited level, we can determine the qubit state by measuring the junction voltage.<sup>4</sup>

Decoherence in *d*-wave qubits is a controversial subject but not central here, so we merely mention a few sources besides external noise (e.g., in  $I_b$ ). The contribution of ungapped nodal quasiparticles is often overestimated: for a misoriented grain boundary, a node on one side always faces a gapped direction on the other, suppressing tunneling exponentially.<sup>20</sup> More problematic are midgap (Andreev) states. Still, since these are split at the qubit's working point, the decoherence due to them can be shown to be tolerable.<sup>21</sup>

As a sideline, a classic double-well system with a tunnel splitting is the  $\text{NH}_3$  molecule. Taking a heavier central nucleus, one arrives at  $\text{PH}_3$  and  $\text{AsH}_3$  as instances of Fig. 1a on a much larger energy scale.<sup>22</sup>

In conclusion, it has been shown that microwave coupling via an auxiliary level suffices for coherent control of a Josephson-phase qubit. The advantages include comparative tolerance to device-parameter spread, ability to operate without refocusing, and speed. Charge-noise tolerance (cf. the first paragraph) should be excellent: without a need for  $\phi$ -tunneling, the ratio of  $E_J$  to the charging energy  $E_C$  can (and should) be comparatively large. A finite  $E_C$  is needed only to ensure appreciable level spacings, as determined by the plasma frequency  $\sim \sqrt{E_J E_C}$ ; suitable device parameters can be readily chosen. For full-fledged quantum computing, one should additionally describe the coupling of these qubits into a quantum register. While, e.g., tunable-bus proposals<sup>23</sup> have the promise of being able to couple any type of Josephson qubit, the detailed investigation is still in progress.

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  - <sup>10</sup> Also,  $\langle 0|A|1\rangle = 0$  for any  $A$ : the qubit wave functions have exponentially small spatial overlap due to the high tunneling barrier. Without this assumption, if e.g.  $E_0 \approx 2E_1$  in regime (b), then the  $\omega_1$  pulse could unintentionally irradiate the  $|0\rangle \leftrightarrow |1\rangle$  as well as the  $|1\rangle \leftrightarrow |2\rangle$  transition.
  - <sup>11</sup> Cf. D.V. Averin, J.R. Friedman, and J.E. Lukens, *Phys. Rev. B* **62**, 11802 (2000), Eq. (22): our off-diagonal  $u_j$  play a role formally analogous to their tunneling amplitude.
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